# Effect of externally applied periodic force on ion acoustic waves in superthermal plasmas

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# Effect of externally applied periodic force on ion acoustic waves in superthermal plasmas

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Ion acoustic solitary waves in superthermal plasmas are investigated in the presence of trapped electrons. The reductive perturbation technique is employed to obtain a forced Korteweg–de Vries-like Schamel equation. An analytical solution is obtained in the presence of externally applied force. The effect of the external applied periodic force is also observed. The effect of the spectral index ($\kappa$), the strength ($f_0$), and the frequency ($\omega$) on the amplitude and width of the solitary wave is obtained. The result may be useful in laboratory plasma as well as space environments. *Published by AIP Publishing.* https://doi.org/10.1063/1.5017559

## I. INTRODUCTION

The existence of highly energetic superthermal particles in different plasma situations, which results in long-tailed distributions, is an essential part in different space[13,29–34] and laboratory plasma[34–40] inspections. A good number of different models have been suggested to relate this effect on nonlinear wave dynamics through phenomenological correction to the electron distribution function.

It is important to note that a way of dealing with non-Maxwellian plasma modeling is given by the kappa distribution[1–3] which was reported by Vasyliunas[1] for the first time to fit phenomenologically the special power law-like dependence of electron distribution functions that are observed in space plasma environments. The spectral index kappa ($\kappa$), for which the distribution is known as the kappa distribution, is acted to modify the effective thermal speed in the case of the distribution function. It is seen that at low values of $\kappa$, the distributions exhibit strong superthermality. This superthermal distribution tends to a Maxwellian distribution as $\kappa$ tends to infinity. Furthermore, it is commonly fitted to observational data.[2–5]

It is well known that the Korteweg–de Vries (KdV) equation describes the weakly nonlinear dispersive waves in small but finite amplitude limit, and this theory has been employed to study the ion acoustic waves in plasmas.[6] Recently, many authors have analysed ion acoustic waves and electrostatic structures in superthermal plasmas by applying the KdV theory.[7,8] Another commonly perceived phenomenon in both space plasma and experimental plasma is that of particle trapping phenomena, whereby few of the plasma particles are imprisoned to a finite area of phase space where they bounce back and forth. These studies have been reported numerically[9,10] and have been found in both space environments and laboratory environments.[11–13] In this paper, we have studied the effect of particle trapping in a $\kappa$ distributed plasma when there is an external periodic perturbation.

The effect of external periodic perturbation has been noted in some real physical situations,[14–16] and it has also been found that these external periodic perturbations may vary on different physical conditions. Few recent studies have emphasized on the study of nonlinear traveling wave solution[18–20] considering an external periodic perturbation. In this paper, our aim is to investigate the ion acoustic solitary waves in an unmagnetized plasma with $\kappa$-distributed trapped electrons in the presence of an external periodic perturbation.

The rest of the paper is organized as follows: The model equations are provided in Sec. II. In Sec. III, we derived the forced Schamel equation. We derived the solitary wave solution of the forced Schamel equation in Sec. IV. Section V represents the numerical simulation and discussion, and conclusions are presented in Sec. VI.

## II. MODEL EQUATIONS

Following Schamel[26] and Williams,[21] we have considered both superthermal and trapped electrons. Actually, the superthermal electrons and trapped electrons are defined in different energy regions; when the energy region lies between $-\sqrt{2\phi} < v < \sqrt{2\phi}$ ($v$ is the velocities and $\phi$ is the electric potential), the electrons are trapped and the corresponding $\kappa$ distribution for the trapped electrons is as follows:

$$f_{e,t}^{k}(v,\phi) = \frac{1}{\sqrt{2\pi}\left(\kappa-\frac{3}{2}\right)^{1/2}}\frac{\Gamma(\kappa)}{\Gamma(\kappa-1/2)} \times \left[1+\beta\left(\frac{v^2/2-\phi}{\kappa-\frac{3}{2}}\right)\right]^{-\kappa} \quad \text{for } E_e \leq 0. \quad (1)$$

This is an extension of Schamel's distribution[22] for Maxwellian trapped electrons. As $\kappa$ tends to infinity, we get back the Schamel's equation. Schamel derived this expression by using the concept of a separatrix to the distribution which separates free electrons from trapped electrons and for the latter a trapped parameter $\beta$ was introduced which measures the inverse temperature of the trapped electrons, and

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for superthermal free electrons, we have assumed the distribution as

$$f_{e,f}^{\kappa}(v,\phi)=\frac{1}{\sqrt{2\pi}\left(\kappa-\frac{3}{2}\right)^{\frac{1}{2}}}\frac{\Gamma(\kappa)}{\Gamma\left(\kappa-\frac{1}{2}\right)}\left(1+\frac{\frac{v^2}{2}-\phi}{\kappa-\frac{3}{2}}\right)^{-\kappa}. \tag{2}$$

($\kappa > 3/2$ is a basic requirement for getting a well-defined value of the characteristic speed.)

So, our $n_e(\phi)$ will be given by

$$n_e(\phi)=\int_{-\infty}^{-\sqrt{2\phi}} f_{e,f}^{\kappa}(v,\phi)dv+\int_{-\sqrt{2\phi}}^{\sqrt{2\phi}} f_{e,t}^{\kappa}(v,\phi)dv + \int_{\sqrt{2\phi}}^{+\infty} f_{e,f}^{\kappa}(v,\phi)dv, \tag{3}$$

where $f_{e,f}^{\kappa}(v,\phi)$ is represented by Eq. (2) and $f_{e,t}^{k}(v,\phi)$ is represented by (1).

After integrating with respect to velocity, we get

$$\begin{aligned} n_e(\phi) &= (2\kappa-3)^{\kappa-3/2}(2\kappa-3-2\phi)^{-\kappa} \\ &\times\left[(2\kappa-3)\sqrt{2\kappa-3-2\phi}-\frac{4}{\Gamma[\kappa-3/2]}\sqrt{2/\pi}\right. \\ &\left.\times\sqrt{\phi}\Gamma[\kappa]2^{F_1}\left[\frac{1}{2},\kappa,\frac{3}{2},\frac{2\phi}{3-2\kappa+2\phi}\right]\right] \\ &+\frac{2}{\Gamma[\kappa-1/2]}\sqrt{2/\pi}(2\kappa-3)^{\kappa-1/2}\sqrt{\phi} \\ &\times(2\kappa-3-2\beta\phi)^{-\kappa}\Gamma[\kappa] \\ &\times 2^{F_1}\left[\frac{1}{2},\kappa,\frac{3}{2},\frac{2\beta\phi}{3-2\kappa+2\beta\phi}\right]. \end{aligned} \tag{4}$$

Keeping in mind that the hyper geometric function $F(a,b,c,x)$ has the following power series expansion $F(a,b,c,x)=1+\frac{ab}{c}x+\frac{a(a+1)b(b+1)}{c(c+1)}\frac{x^2}{2!}+\cdots$ (keeping term up to $\phi^2$), the expansion function $n_e(\phi)$:

$$\begin{aligned} n_e(\phi) &\sim 1+\left(\frac{2\kappa-1}{2\kappa-3}\right)\phi+\frac{8\sqrt{2/\pi}(\beta-1)\kappa\Gamma[\kappa]}{3(2\kappa-3)^{3/2}\Gamma[\kappa-1/2]}\phi^{3/2} \\ &+\frac{4\kappa^2-1}{2(2\kappa-3)^2}\phi^2 \\ n_e(\phi) &\sim 1+p\phi+q\phi^{3/2}+r\phi^2, \end{aligned} \tag{5}$$

where $p=\frac{2\kappa-1}{2\kappa-3}$, $q=\frac{8\sqrt{2/\pi}(\beta-1)\kappa\Gamma(\kappa)}{3(2\kappa-3)^{3/2}\Gamma(\kappa-1/2)}$, and $r=\frac{4\kappa^2-1}{2(2\kappa-3)^2}$.

The one dimensional normalized fluid equations[21] are given by

$$\frac{\partial n}{\partial t}+\frac{\partial(nu)}{\partial x}=0, \tag{6}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=-\frac{\partial \phi}{\partial x}, \tag{7}$$

$$\begin{aligned} \frac{\partial^2\phi}{\partial x^2} &= n_e-n+S(x,t) \\ &\simeq -(n-1)+p\phi+q\phi^{3/2}+r\phi^2+S(x,t), \end{aligned} \tag{8}$$

where $p=\frac{2\kappa-1}{2\kappa-3}$, $q=\frac{8\sqrt{2/\pi}(\beta-1)\kappa\Gamma(\kappa)}{3(2\kappa-3)^{3/2}\Gamma(\kappa-1/2)}$, and $r=\frac{4\kappa^2-1}{2(2\kappa-3)^2}$.

Here, $n$, $u$, $\phi$ represent the ion density, velocity, and electrostatic potential, respectively. The term $S(x, t)$ is a charge density source arising from experimental conditions for a single definite purpose. Now, we consider the following normalizations: lengths are normalized by Debye length $\lambda_D=\left(\frac{\epsilon_0 k_B T_e}{n_0 Z e^2}\right)^{1/2}$, time ($t$) by the inverse of the plasma frequency $\omega_p=\left(\frac{n_0 Z^2 e^2}{\epsilon_0 m}\right)^{1/2}$, number density by the equilibrium ion density $n_0$, electrostatic potential by $\left(\frac{k_B T_e}{e}\right)$, and velocities by the characteristic speed $c_s=\left(\frac{Z k_B T_e}{m^{1/2}}\right)$.

## III. DERIVATION OF THE FORCED SCHAMEL EQUATION

To study the nonlinear phenomena, we introduce the reductive perturbation technique (RPT) of Schamel.[22] According to RPT, the stretch coordinates as follows:

$$\zeta=\epsilon^{1/4}(x-vt), \tag{9}$$

$$\tau=\epsilon^{3/4}t, \tag{10}$$

where $\epsilon$ is an infinitely small parameter. The dependent variables $n, u, \phi, S(x,t)$ can be expanded as follows:

$$n\sim 1+\epsilon n_1+\epsilon^{3/2}n_2+\cdots, \tag{11}$$

$$u\sim \epsilon u_1+\epsilon^{3/2}u_2+\cdots, \tag{12}$$

$$\phi\sim \epsilon \phi_1+\epsilon^{3/2}\phi_2+\cdots, \tag{13}$$

$$S(x,t)\sim \epsilon^{3/2}S_2(x,t)+\cdots. \tag{14}$$

Substituting Eqs. (9)–(14) in the model equations (6)–(8) and comparing the coefficients of different order of $\epsilon$, one can obtain

$$\epsilon: n_1=p\phi_1, \tag{15}$$

$$\epsilon^{3/2}: \frac{\partial^2\phi_1}{\partial\xi^2}+n_2=p\phi_2+q\phi_1^{3/2}+S_2(\xi,\tau), \tag{16}$$

$$\epsilon^{5/4}: -v\frac{\partial n_1}{\partial\xi}+\frac{\partial u_1}{\partial\xi}=0,\quad v\frac{\partial u_1}{\partial\xi}-\frac{\partial\phi_1}{\partial\xi}=0, \tag{17}$$

$$\epsilon^{7/4}: -v\frac{\partial n_2}{\partial\xi}+\frac{\partial u_2}{\partial\xi}+\frac{\partial n_1}{\partial\tau}=0,\quad -v\frac{\partial u_2}{\partial\xi}+\frac{\partial u_1}{\partial\tau}+\frac{\partial\phi_2}{\partial\xi}=0. \tag{18}$$

To obtain dispersion relation, we eliminate the perturbed quantities from (15) and (16) and we get

$$v^2=\frac{1}{p}. \tag{19}$$

The detailed calculation is shown in the Appendix.

Considering Eqs. (15)–(18), we get the nonlinear evolution equation as

$$\frac{\partial \phi_1}{\partial \tau} + A\sqrt{\phi_1}\frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} = B\frac{\partial S_2}{\partial \xi}, \tag{20}$$

where

$$A = -\frac{3q}{4p^{3/2}}, \quad B = \frac{1}{2p^{3/2}}.$$

Recently, Sen *et al.*[17] have studied non-linear wave excitation by orbiting charged space debris objects. They considered the source term $S(x - v_d t)$ in the Poisson equation which is obtained from the charged debris, moving at a speed $v_d$. There is no work in the area where the externally applied force is solely dependent on $\tau$. Recently, Ali *et al.*[27] and Saha and Chatterjee[28] have considered the same (the source term) in the Poisson equation. Being motivated by these works, we have considered the source term on the Poisson equation, instead of equation of continuity; however, till today no work has been reported in the form of KdV-like Schamel equation. Considering their works, we suppose $S_2 = \frac{f_0}{B}\xi\cos(\omega\tau)$, where $f_0$ and $\omega$ denote the strength and frequency, respectively. Using this, one can obtain from Eq. (20)

$$\frac{\partial \phi_1}{\partial \tau} + A\sqrt{\phi_1}\frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} = f_0\cos(\omega\tau), \tag{21}$$

and this is known as forced KdV-like Schamel equation.

## IV. SOLITARY WAVE SOLUTION OF THE FORCED SCHAMEL EQUATION

The Schamel equation with an external periodic force $f_0\cos(\omega\tau)$ is

$$\frac{\partial \phi_1}{\partial \tau} + A\sqrt{\phi_1}\frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} = f_0\cos(\omega\tau), \tag{22}$$

where $f_0$ is the strength of the periodic force and $\omega$ is its frequency.

When $f_0 = 0$, Eq. (22) represents KdV-like Schamel equation and the solitary wave solution is of the form

$$\phi_1 = \phi_m \operatorname{sech}^4\left(\frac{\xi - U\tau}{W}\right), \tag{23}$$

where $\phi_m = \left(\frac{15U}{8A}\right)^2$ and $W = \sqrt{\frac{16B}{U}}$ are the amplitude and width of the ion-acoustic solitary wave, respectively, and $U$ is the speed of the ion-acoustic solitary wave.

It is known that for a KdV equation

$$I = \int_{-\infty}^{\infty} \phi_1^2 d\xi \tag{24}$$

is a conserved quantity.

It can be easily shown that $I$ is conserved in the case of the Schamel equation also.

When $f_0 \neq 0$, we consider the amplitude, width, and velocity of the solitary wave dependent on $\tau$[23–25] and the approximate solution of (22) is of the form

$$\phi_1 = \phi_m(\tau)\operatorname{sech}^4\left(\frac{\xi - U(\tau)\tau}{W(\tau)}\right), \tag{25}$$

where the amplitude $\phi_m(\tau) = \left(\frac{15U(\tau)}{8A}\right)^2$ and the width $W(\tau) = \sqrt{\frac{16B}{U(\tau)}}$, $U(\tau)$ have to be determined.

Considering the same as the conservation law of KdV equation, one can obtain

$$I = \frac{10125}{224A^4}\sqrt{B}U(\tau)^{7/2}. \tag{26}$$

Also

$$\int_{-\infty}^{\infty} \phi_1 d\xi = \frac{4}{3}\phi_m(\tau)W(\tau) = \frac{75}{4}\frac{U(\tau)^{3/2}\sqrt{B}}{A^2}. \tag{27}$$

Differentiating (24) with respect to $\tau$

$$\frac{dI}{d\tau} = 2f_0\cos(\omega\tau)\int_{-\infty}^{\infty}\phi_1 d\xi \left(\text{using boundary conditions that } \phi_1 \text{ and } \frac{\partial \phi_1}{\partial \xi} \text{ vanished when } \xi \to \pm\infty\right). \tag{28}$$

Using (26) and (27) in Eq. (28), we have

$$U(\tau)\frac{d}{d\tau}U(\tau) = \frac{32A^2}{135}f_0\cos(\omega\tau). \tag{29}$$

Integrating (29) with respect to $\tau$, we have

$$U(\tau)^2 = \frac{64A^2}{135}\frac{f_0\sin(\omega\tau)}{\omega} + M_0. \tag{30}$$

Initially, $\tau = \tau_0$, $M_0 = k^2$.

So

$$\begin{aligned} U(\tau)^2 &= \frac{64A^2}{135}\frac{f_0\sin(\omega t)}{\omega} + k^2 \\ U(\tau) &= \sqrt{\frac{64A^2}{135}\frac{f_0\sin(\omega t)}{\omega} + k^2}. \end{aligned} \tag{31}$$

So, the solution of (22) is of the form

$\phi = \phi_m(\tau)\sec h^4\left(\frac{\xi - U(\tau)\tau}{W(\tau)}\right)$, where $U(\tau)$ is given by (31),

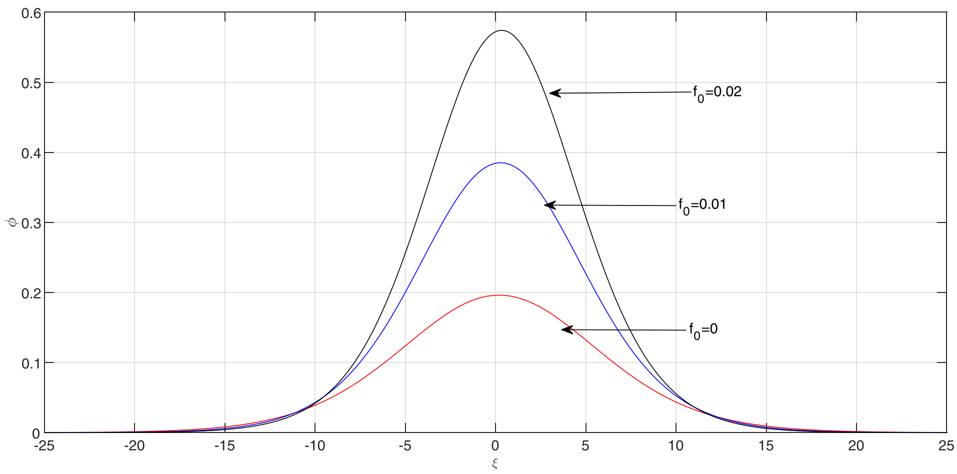


FIG. 1. Variation of the solitary wave solution of Eq. (22) for $f_0 = 0$ (red curve), $f_0 = 0.01$ (blue curve), and $f_0 = 0.02$ (black curve) with $\kappa = 2.5$, $U_0 = 0.2, \beta = 0.5, \omega = 1, \tau = 1$.

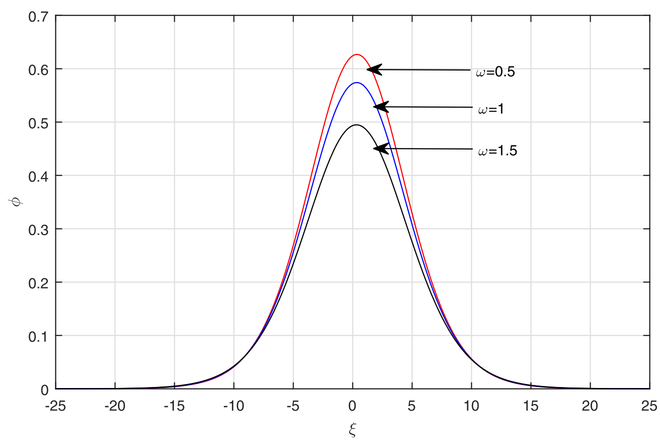


FIG. 2. Variation of the solitary wave solution of Eq. (22) for $\omega = 0.5$ (red curve), $\omega = 1$ (blue curve), and $\omega = 1.5$ (black curve) with $\kappa = 2.5$, $U_0 = 0.1, \beta = 0.5, f_0 = 0.02, \tau = 1$.

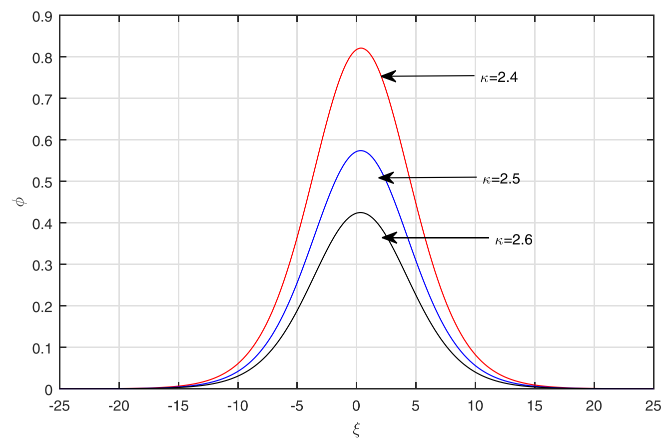


FIG. 3. Variation of the solitary wave solution of Eq. (22) for $\kappa = 2.4$ (red curve), $\kappa = 2.5$ (blue curve), and $\kappa = 2.6$ (black curve) with other parameters same as in Fig. 2.

and the amplitude and width are as follows:

$$\phi_m(\tau) = \frac{5f_0 \sin(\omega\tau)}{3\omega} + \frac{225k^2}{64A^2}$$

$$W(\tau) = \left(\frac{16B}{\sqrt{\frac{64A^2 f_0 \sin(\omega\tau)}{135\omega} + k^2}}\right)^{1/2}.$$

## V. NUMERICAL SIMULATION AND DISCUSSIONS

The effects of parameters $f_0, \omega$, spectral index $\kappa$ on the ion acoustic solitary wave structure of the KdV like Schamel equation (22) have been studied in this section.

Figure 1 shows the variation of the ion acoustic solitary wave for different values of $f_0$ with other parameters $\kappa = 2.5$, $U_0 = 0.2$, $\beta = 0.5$, $\omega = 1$, $\tau = 1$. It is observed that the amplitude of the ion acoustic solitary wave increases as the strength $f_0$ of the external periodic force increases.

In Fig. 2, we present the variation of the ion acoustic solitary wave for different values of frequency $\omega$ of the externally applied force with fixed values of other parameters $\kappa = 2.5$, $U_0 = 0.1$, $\beta = 0.5$, $f_0 = 0.02$, $\tau = 1$. It is seen that the amplitude of the ion acoustic solitary wave decreases as the frequency of the external periodic force increases.

Figure 3 reflects the variation of the ion acoustic solitary wave for different values of the spectral index ($\kappa$) with fixed values of other parameters $\omega = 1$, $U_0 = 0.1$, $\beta = 0.5$, $f_0 = 0.02$, $\tau = 1$. It is observed that as the spectral index ($\kappa$) increases, the amplitude of the ion acoustic solitary waves decreases.

In Fig. 4, we plot $\phi_m(\tau)$ vs. $f_0$ for different values of frequency $\omega$ of the external periodic perturbation, and other

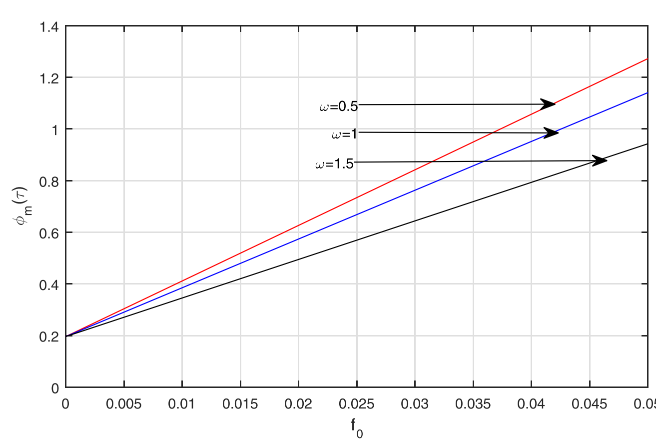


FIG. 4. Variation of the Amplitude of the solitary wave solution of the equation (1) for $\omega$=0.5 (red curve), $\omega$=1 (blue curve) and $\omega$=1.5 (black curve) with other parameters same as Fig. 2.

parameters are same as in Fig. 2. The amplitude of the solitary wave increases as the force increases. At the same time, when the frequency of the external periodic forces increases, the rate of change of amplitude of the solitary wave decreases.

## VI. CONCLUSIONS

We have studied ion acoustic solitary waves in superthermal plasmas in the presence of trapped electrons. The reductive perturbation technique has been employed to derive the KdV-like Schamel equation. An analytical solitary wave solution has been derived for the Schamel equation in the presence of the externally applied periodic force. The effect of the externally applied periodic force on the ion acoustic solitary wave solution with fixed values of other physical parameters $\kappa$, $U_0$, $\beta$ has been presented. The solitary wave becomes smooth when the strength ($f_0$) of the external force decreases. On the other hand, the amplitude of the solitary wave increases when the frequency ($\omega$) of the periodic force decreases. The result may be useful in laboratory plasmas as well as space environments.

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## APPENDIX: SOLITARY WAVE SOLUTION OF THE FORCED SCHAMEL EQUATION OF EQ. (29)

$$I = \int_{-\infty}^{\infty} \phi_1^2 d\xi = \int_{-\infty}^{\infty} \phi_m^2(\tau)\,\sec h^8\left(\frac{\xi - U(\tau)\tau}{W(\tau)}\right) d\xi. \quad \text{(A1)}$$

Now

$$\int_{-\infty}^{\infty} \operatorname{sech}^8\left(\frac{\xi - U(\tau)\tau}{W(\tau)}\right) d\xi = \frac{32W(\tau)}{35}$$

$$\begin{aligned} I &= \phi_m^2(\tau)\frac{32W(\tau)}{35} \\ &= \frac{10125}{224A^4}\sqrt{B}U(\tau)^{7/2}. \end{aligned} \quad \text{(A2)}$$

Also

$$\int_{-\infty}^{\infty} \phi_1 d\xi = \frac{4}{3}\phi_m(\tau)W(\tau) = \frac{75}{4}\frac{U(\tau)^{3/2}\sqrt{B}}{A^2}. \quad \text{(A3)}$$

Differentiating (A1) with respect to $\tau$

$$\begin{aligned} \frac{dI}{d\tau} &= \int_{-\infty}^{\infty} 2\phi_1 \frac{\partial \phi_1}{\partial \tau} d\xi \\ &= \int_{-\infty}^{\infty} 2\phi_1\left(-A\sqrt{\phi_1}\frac{\partial\phi_1}{\partial\xi} - B\frac{\partial^3\phi_1}{\partial\xi^3} + f_0\cos(\omega\tau)\right) d\xi \\ \frac{dI}{d\tau} &= 2f_0\cos(\omega\tau)\int_{-\infty}^{\infty}\phi_1 d\xi \left(\textit{using boundary conditions that } \phi_1 \textit{ and } \frac{\partial\phi_1}{\partial\xi} \textit{ vanished when } \xi \to \pm\infty\right). \end{aligned} \quad \text{(A4)}$$

Using (A2) and (A3) in Eq. (A4), we have

$$\begin{aligned} \frac{d}{d\tau}\left(\frac{10125}{224A^4}\sqrt{B}U(\tau)^{\frac{7}{2}}\right) &= 2\frac{75}{4}\frac{U(\tau)^{3/2}B^{1/2}}{A^2}f_0\cos(\omega\tau) \\ \Rightarrow \frac{10125}{224A^4}\sqrt{B}U(\tau)^{\frac{5}{2}}\frac{d}{d\tau}U(\tau) &= \frac{75}{2}\frac{U(\tau)^{3/2}B^{1/2}}{A^2}f_0\cos(\omega\tau) \\ U(\tau)\frac{d}{d\tau}U(\tau) &= \frac{32A^2}{135}f_0\cos(\omega\tau). \end{aligned}$$


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